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Distributed Formation Flight Control Using Constraint Forces

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A new approach for formation flight control of multiple aircraft is presented. Constraint forces are used to derive the dynamics of a constrained, multibody system. A stable, distributed control algorithm is designed based on the information flow graph for a group of aircraft. The aircraft will achieve a particular formation while ensuring an arbitrarily small bounded navigation tracking error with parameter uncertainties of the entire group. It is assumed that uncertainty exists in the drag coefficient of each aircraft. An adaptation algorithm is developed to compensate for the uncertainty and estimate the drag coefficient. The advantage of the proposed distributed control algorithm is that it allows the addition/removal of other aircraft into/from the formation seamlessly with simple modifications of the control input. Furthermore, the algorithm provides inherent scalability. Simulations were conducted to verify the proposed approach.

I. Introduction

THE problem of autonomous formation flight is an important research area in the aerospace field. Multiple aircraft flight in formations with defined geometries leads to many advantages and applications, for example, energy saving from vortex forces [1] and fuel efficiency via induced drag reduction [2]. Formation flying can also be used for airborne refueling and quick deployment of troops and vehicles. Moreover, many aircraft involved in a mission can be better managed if they fly in a specific formation rather than in an undefined structure.

The main goal of formation flight of multiple aircraft is to achieve a desired group formation shape while controlling the overall behavior of the group. Most of the formation flight strategies consist of variations in the leader/wingman formation [3–5], that is, a lead aircraft is chosen to direct the formation and all other wingmen are expected to maintain a fixed relative distance with respect to the lead aircraft. Formation control of linear aircraft models using a proportional-integral-derivative controller has been studied by Proud et al. [3]. In Giulietti et al. [4], two different leader/wingman structures were presented: leader mode and front mode. In the leader mode, each wingman takes the trajectory reference from the leader, whereas, in the front mode, each aircraft takes its reference from the preceding aircraft. To overcome the limitation of the leader/wingman strategy, Giulietti et al. [6] later proposed a strategy in which each aircraft is not required to keep its position with respect to the formation leader, but an imaginary point in the formation. In Singh et al. [5], invertibility of input–output maps and control system design for nonlinear formation flying were considered. In addition to research on aircraft formation flight, there have also been a number of studies on coordinating multiple mobile robots and spacecraft. Although the applications are different, the fundamental approach to coordination of multiple aircraft, mobile robots, and spacecraft is similar. Numerous control schemes have been proposed for the multi-agent coordination problem. Several surveys of the literature in cooperative control have taken a broad view (see, for instance, Murray [7] and references therein).

A popular approach to achieve coordination of multiple vehicles is to consider the mechanical nature of the vehicles and shape the dynamics of the formation using potential fields. Coordinated control using potential functions can be found in Leonard and Fiorello [8], Olfati-Saber and Murray [9], and Ogren et al. [10]. The basic idea in these studies is to create an energy-like function (potential function) in terms of the distance constraints between vehicles and use the negative gradient of the potential function as a restoring force on each vehicle to achieve coordination. In Leonard and Fiorello [8], an approach for distributed control of multiple agents by using artificial potential functions and virtual leaders was given. Each individual agent behaves according to the interaction forces generated by sensing the positions of neighboring agents. In addition to the intervehicle potential fields, local potential fields associated with virtual leaders are introduced. In Olfati-Saber and Murray [9], a specific potential function, which is a function of the distance constraints of the desired formation, is used. Formation stabilization and tracking for multiple vehicles is also considered in Olfati-Saber and Murray [11], where a separation principle that decouples structural stabilization from navigational tracking is applied. The idea of artificial potential functions for obstacle avoidance is applied to robot navigation and control in Rimon and Koditschek [12]. Extension of the work of Rimon and Koditschek [12] to multiple vehicles with kinematic models can be found in Dimarogonas et al. [13]. Instead of relying on repelling potential forces, Chang and Marsden [14] presented a control law for multiple systems based on gyroscopic forces for collision and obstacle avoidance; the gyroscopic forces are used for obstacle avoidance without affecting the global potential function. An approach called the virtual structure method for spacecraft formation flying can be found in [15].

The new approach for formation flight of multiple aircraft presented in this paper uses the theory of constraint forces to build a formation from arbitrary initial conditions for the aircraft. The idea of constrained dynamics for a system of multiple bodies with constraints is that the description of the system not only includes the external forces acting on the bodies, but also the constraint forces which limit the motion of the system to be consistent with the constraints. The constraints on the system are imposed by adding a set of forces to the governing equations, which keep formation separation constraints satisfied for all time [16,17]. The key idea of the proposed work is to use the theory of constraint forces to

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determine the total force required on each aircraft to maintain the formation separation. The force required to maintain the constraints for formation of a group of aircraft is calculated directly. A centralized control strategy with full information for formation of a group of vehicles using the notion of constraint forces was given in Zou and Pagilla [18]. This paper describes a scalable, distributed control strategy for aircraft formation.

In the potential (or penalty) function approach, the square of the constraint function (or some other appropriate positive function of constraints) is treated as a potential energy. A formation-keeping force that is proportional to the gradient of the potential energy is used. Because these restoring forces, which rely on displacements, are regular forces, they compete with every other applied force. The advantage of the constraint force approach is that the calculated constraint forces cancel only those applied forces that act against the constraints. The main contribution of this paper is in the development of a stable, scalable, and distributed control algorithm for multiple aircraft using constraint forces that will simultaneously achieve, and maintain, a given formation together with tracking of a desired group trajectory. Moreover, the control algorithm is adaptive in the sense that the uncertain drag coefficient of each aircraft is estimated by an adaptive algorithm.

The rest of the paper is organized as follows. Section II gives the nonlinear model for each aircraft. In Sec. III, the distributed constraint force approach for the coordination of multiple aircraft is developed. First, to illustrate the method, a stable control algorithm that will achieve and maintain a desired distance between two aircraft is designed based on the virtual constraint force between them. Then, a stable, distributed control algorithm is proposed for an arbitrary number of aircraft with a given information flow pattern between aircraft. Section IV gives simulation results on an example of three aircraft. Conclusions and future work are given in Sec. V.

II. Aircraft Dynamics and Control System Structure

The nonlinear equations of motion for an aircraft over a flat, nonrotating Earth are modeled by 12 state equations [19]. A common control system strategy for an aircraft is a two-loop structure where the attitude dynamics are controlled by an inner loop, and the position dynamics are controlled by an outer loop. In the context of a group of aircraft in formation, the outer loop also contains a controller that can achieve and maintain the given formation. A schematic of such a two-loop strategy is shown in Fig. 1. The outer-loop controller is designed to track the given position commands and reach the desired formation shape, using the lift L_i, bank angle μ_i, and engine thrust T_i as the control inputs. The inner-loop controller is designed to follow the desired attitude generated from the outer loop, using the virtual control surface deflections of the aileron, elevator, and rudder.

The inner-loop design to control angular rates using control surface deflections has been well studied in the literature [20–22]. For this reason, the outer-loop design, including the formation controller, is focused on in this study. It is assumed that the inner loop is well designed to regulate the attitude of the aircraft. Therefore, the following dynamic model containing only those state variables that pertain to the outer-loop design is considered:

$$\dot{x}_i = V_i \cos \chi_i \cos \gamma_i \tag{1}$$

$$\dot{y}_i = V_i \sin \chi_i \cos \gamma_i \tag{2}$$

$$\dot{h}_i = V_i \sin \gamma_i \tag{3}$$

$$\dot{V}_i = -g \sin \gamma_i + \frac{1}{m_i}(T_i - D_i) \tag{4}$$

$$\dot{\chi}_i = \frac{L_i \sin \mu_i}{m_i V_i \cos \gamma_i} \tag{5}$$

$$\dot{\gamma}_i = \frac{1}{m_i V_i}(L_i \cos \mu_i - m_i g \cos \gamma_i) \tag{6}$$

A group of n aircraft is considered, that is, $i = 1, 2, \ldots, n$. The coordinates x_i and y_i and the altitude h_i specify the position of the center of gravity of the ith aircraft in an Earth-based reference frame. The orientation of the aircraft, that is, the direction of the velocity vector, is denoted by the heading angle χ_i, flight-path angle γ_i, and bank angle μ_i. The heading angle is the angle between the projection of the velocity vector onto the xy plane and the x axis. The angle between the velocity vector and its projection onto the xy plane is the flight-path angle. The bank angle is the angle between the aircraft lift and weight force vectors. The aircraft velocity V_i is assumed to be equal to the airspeed. In Fig. 2, T_i is the engine thrust, D_i is the drag, L_i is the lift, m_i is the aircraft mass, and g is the acceleration due to gravity. The thrust depends on the altitude h_i, velocity V_i, and the throttle setting η_i by a known relationship $T_i = T_i(h_i, V_i, \eta_i)$. Also, it is assumed that the drag is a function of h_i, V_i, and L_i, that is, $D_i = D_i(h_i, V_i, L_i)$.

In the model, the engine thrust T_i, the lift L_i, and the bank angle μ_i are the control variables for the aircraft. The drag can be expressed as a function of a nondimensional drag coefficient C_{D_i} in the following form:

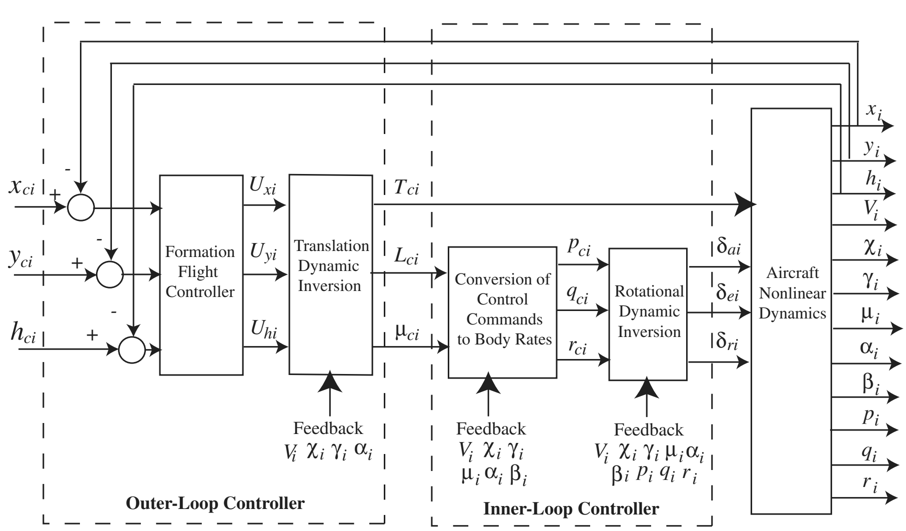

Fig. 1 Flight control system structure

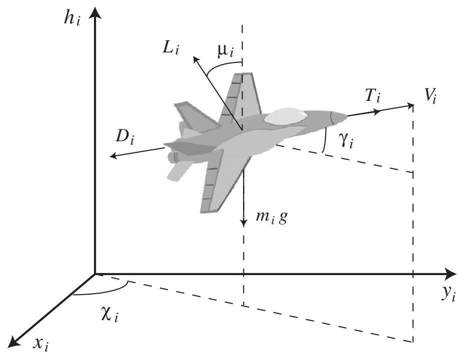

Fig. 2 Three-dimensional geometry used to define variables in aircraft model.

$$D_i = \tfrac{1}{2}\rho V_i^2 S_i C_{D_i} \tag{7}$$

where S_i is an aerodynamic reference area of the aircraft, and the quantity ρ is the average density of air. Alternatively, the air density could be considered as a function of altitude of aircraft. For simplicity, the air density is assumed to be a constant. The drag coefficient is assumed to have a nominal component and a component which increases quadratically with the lift as given by

$$C_{D_i} = C_{D_{0i}} + K_i C_{L_i}^2 \tag{8}$$

where $C_{D_{0i}}$ is the profile drag coefficient, which is assumed to be a constant, C_{L_i} is the lift coefficient, and $K_i C_{L_i}^2$ is the induced drag. Typical values for $C_{D_{0i}}$ for the whole aircraft are on the order of 0.003–0.02. Replacing the lift coefficient with the load factor, the drag D_i can be computed as

$$D_i = \frac{1}{2}\rho V_i^2 S_i C_{D_{0i}} + 2K_i \frac{L_i^2}{\rho V_i^2 S_i} \tag{9}$$

It is assumed that aggressive maneuvering will not be necessary to hold a desired formation and the aircraft will operate close to wing level, steady-state flight. Therefore, any uncertainties in drag forces will dominate and be most influential to the aircraft dynamics. To compensate for uncertainties in drag, the coefficient $C_{D_{0i}}$ is considered to be an unknown parameter and is estimated with an adaptive law.

Differentiating Eqs. (1–3) with respect to time, and substituting dynamics of V_i, χ_i, and γ_i from Eqs. (4–6), the dynamics of the position of the aircraft are given by

$$\ddot{x}_i = U_{x_i} - \frac{\rho V_i^2 S_i C_{D_{0i}}}{2m_i} \cos\chi_i \cos\gamma_i \tag{10}$$

$$\ddot{y}_i = U_{y_i} - \frac{\rho V_i^2 S_i C_{D_{0i}}}{2m_i} \sin\chi_i \cos\gamma_i \tag{11}$$

$$\ddot{h}_i = U_{h_i} - \frac{\rho V_i^2 S_i C_{D_{0i}}}{2m_i} \sin\gamma_i \tag{12}$$

where U_{x_i}, U_{y_i}, and U_{h_i} are the virtual control variables. If the virtual control variables are known, the actual control variables can be obtained using the following expressions [23]:

$$\mu_{c_i} = \arctan\left(\frac{U_{y_i}\cos\chi_i - U_{x_i}\sin\chi_i}{\cos\gamma_i(U_{h_i}+g) - \sin\gamma_i(U_{x_i}\cos\chi_i + U_{y_i}\sin\chi_i)}\right) \tag{13}$$

$$L_{c_i} = m_i \frac{\cos\gamma_i(U_{h_i}+g) - \sin\gamma_i(U_{x_i}\cos\chi_i + U_{y_i}\sin\chi_i)}{\cos\mu_{c_i}} \tag{14}$$

$$T_{c_i} = m_i[\sin\gamma_i(U_{h_i}+g) + \cos\gamma_i(U_{x_i}\cos\chi_i + U_{y_i}\sin\chi_i)] + 2K_i \frac{L_{c_i}^2}{\rho V_i^2 S_i} \tag{15}$$

which is valid for bank angle commands, $-\pi/2 < \mu_c < \pi/2$. A four-quadrant arctan function can be used to ensure a one-to-one mapping of Eq. (13).

The prelinearized aircraft models of Eqs. (10–12) can be expressed in the following form:

$$\ddot{q}_i = U_i + \Delta_i \tag{16}$$

where $q_i = [x_i, y_i, h_i]^T \in \mathbb{R}^3$ is the position, and $U_i = [U_{x_i}, U_{y_i}, U_{h_i}]^T \in \mathbb{R}^3$ is the control input. The uncertain quantity Δ_i in Eq. (16) can be expressed as $\Delta_i = \Psi_i C_{D_{0i}}$ where $\Psi_i \in \mathbb{R}^{3\times 1}$ is a known function, given by

$$\Psi_i = \begin{bmatrix} -\frac{\rho V_i^2 S_i}{2m_i}\cos\chi_i\cos\gamma_i \\ -\frac{\rho V_i^2 S_i}{2m_i}\sin\chi_i\cos\gamma_i \\ -\frac{\rho V_i^2 S_i}{2m_i}\sin\gamma_i \end{bmatrix} \tag{17}$$

Based on the prelinearized aircraft dynamics given by Eq. (16) for each of the n aircraft, the goal is to design a distributed formation controller that achieves and maintains a given formation, together with tracking of a desired group trajectory under a given information flow between different aircraft within the group.

III. Formation Controller Design

The formation structural topology of the aircraft group can be defined as a formation graph, which can be used to study the relative position of aircraft in the group by applying graph theory [24]. The formation graph of n aircraft is defined as an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, \ldots, n\}$ is a finite set of vertices (nodes) in correspondence with the n aircraft in the group, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of edges (i, j) representing interaircraft position specifications. The neighborhood set of the aircraft i, $\mathcal{N}_i = \{j | (i, j) \in \mathcal{E}, j \neq i\}$, includes all other aircraft which communicate with it. For simplicity, the information flow graph and the formation graph are assumed to be identical.

In the constraint force approach, geometric constraints are imposed on the system of bodies (aircraft in this case) by adding a set of constraint forces to the governing equations that keep the constraints satisfied. The overall control input U_i for the ith aircraft, which is required to achieve/maintain the formation and track the desired group trajectory, can be expressed as

$$U_i = F_i + F_{c_i} \tag{18}$$

where F_i is the applied force per unit mass, and F_{c_i} is the constraint force per unit mass that limits the motion of the system to be consistent with the constraints. To compensate for the uncertainties in the model, the applied force F_i is written as

$$F_i = F_{n_i} - F_{\mathrm{ad}_i} \tag{19}$$

where F_{n_i} is the navigational feedback control given as

$$F_{n_i} = \ddot{q}_{d_i} - c_1 e_i - c_2 \dot{e}_i \tag{20}$$

where c_1 and c_2 are positive constants, $e_i = q_i - q_{d_i}$ and $\dot{e}_i = \dot{q}_i - \dot{q}_{d_i}$ are navigational tracking errors, and q_{d_i}, $\dot{q}_{d_i}$, and $\ddot{q}_{d_i}$ are the desired position, velocity, and acceleration, respectively. The adaptive term F_{ad_i} is used to compensate for the uncertainties and is given by

$$F_{\mathrm{ad}_i} = \Psi_i \hat{C}_{D_{0i}} \tag{21}$$

where $\hat{C}_{D_{0i}}$ is the estimate of $C_{D_{0i}}$.

First, the constraint force between a pair of communicating aircraft will be designed. The application of such a force on each aircraft, in addition to the applied force (both navigation and adaptive), will ensure that the constraint is satisfied between the two aircraft. Second, the distributed control algorithm for the entire group will be developed based on the constraint force between a pair of communicating aircraft.

A. Constraint Force Between a Pair of Aircraft

Consider a pair of aircraft i and j that share an edge in the information flow graph, that is, $(i, j) \in \mathcal{E}$. Denote the constraint corresponding to the (i, j) edge in the formation graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ by $\phi_{ij}(q_i, q_j) = 0$ for any $(i, j) \in \mathcal{E}$ and $i \neq j$, with $\phi_{ij}(q_i, q_j)$ in some specific form. Define the composite position vector of two communicating aircraft by $q_{ij} = [q_i^T, q_j^T]^T \in \mathcal{R}^{6\times 1}$. The constraint function is defined as a function of distance between the two aircraft $\| q_i - q_j \|$,

$$\phi_{ij}(q_{ij}) = z(\| q_i - q_j \|, d_{ij}), \qquad (i, j) \in \mathcal{E} \tag{22}$$

where d_{ij} is the length of the edge (i, j), which is the desired distance between the two aircraft in the formation. The structural constraint for this pair of aircraft can be expressed as

$$\phi_{ij}(q_{ij}) = 0 \tag{23}$$

Differentiating Eq. (22) once, we get the constraint velocity as

$$\dot{\phi}_{ij}(q_{ij}, \dot{q}_{ij}) = \frac{\partial \phi_{ij}(q_{ij})}{\partial q_{ij}} \dot{q}_{ij} := A_{ij}(q_{ij})\dot{q}_{ij} \tag{24}$$

where

$$A_{ij}(q_{ij}) = \frac{\partial \phi_{ij}(q_{ij})}{\partial q_{ij}}$$

is a specially structured 1×6 matrix called the constraint matrix, which is in the form of

$$A_{ij}(q_{ij}) = [\, a_{ij}^T \quad a_{ji}^T \,] \quad \text{with} \quad a_{ij} = -a_{ji} \in \mathbb{R}^3 \tag{25}$$

For example, if the constraint function is defined as a function of the Euclidean distance between two aircraft, $\phi_{ij}(q_{ij}) = \| q_i - q_j \| - d_{ij}$, $(i, j) \in \mathcal{E}$, then the constraint matrix is given by

$$A_{ij}(q_{ij}) = \begin{bmatrix} \frac{(q_i - q_j)^T}{\|q_i - q_j\|} & -\frac{(q_i - q_j)^T}{\|q_i - q_j\|} \end{bmatrix}$$

Differentiating $\dot{\phi}_{ij}(q_{ij}, \dot{q}_{ij})$ again, we get the constraint acceleration as

$$\ddot{\phi}_{ij}(q_{ij}, \dot{q}_{ij}, \ddot{q}_{ij}) = \dot{A}_{ij}(q_{ij}, \dot{q}_{ij})\dot{q}_{ij} + A_{ij}(q_{ij})\ddot{q}_{ij} \tag{26}$$

where

$$\dot{A}_{ij}(q_{ij}, \dot{q}_{ij}) = \frac{\partial \dot{\phi}_{ij}(q_{ij}, \dot{q}_{ij})}{\partial q_{ij}}$$

Assuming that the configuration q_{ij} and the velocity $\dot{q}_{ij}$ both have the desired initial values, that is, $\phi_{ij}(q_{ij}^0) = \dot{\phi}_{ij}(q_{ij}^0, \dot{q}_{ij}^0) = 0$, the velocities and accelerations, respectively, that are consistent with the constraints are given by

$$\dot{\phi}_{ij}(q_{ij}, \dot{q}_{ij}) = 0 \tag{27}$$

$$\ddot{\phi}_{ij}(q_{ij}, \dot{q}_{ij}, \ddot{q}_{ij}) = 0 \tag{28}$$

That is, if the subsystem of the two aircraft i and j begins its motion at the position and velocity that is initially consistent with the constraint, subsequent motion of the two aircraft satisfies the position and velocity constraints if we ensure that the acceleration constraint Eq. (28) is satisfied. Now, the natural question is, how do we ensure that the acceleration constraint equation is met for all time? The answer lies in finding the forces required to maintain the constraints, which are called the constraint forces.

Based on the previous discussion of the notion of the constrained dynamics, using Eqs. (16) and (18–20) for the ith and jth vehicles, the dynamics of the constrained system for the ijth pair of vehicles can be written as

$$\ddot{q}_{ij} = F_{n_{ij}} + F_{c_{ij}} + (\Delta_{ij} - F_{\mathrm{ad}_{ij}}) \tag{29}$$

where $F_{n_{ij}} = [F_{n_i}^T, F_{n_j}^T]^T$, $F_{c_{ij}} = [F_{c_i}^T, F_{c_j}^T]^T$, $F_{\mathrm{ad}_{ij}} = [F_{\mathrm{ad}_i}^T, F_{ad_j}^T]^T$, and $\Delta_{ij} = [\Delta_i, \Delta_j]^T$.

Substituting the constrained system dynamics (29) into Eq. (26), we get

$$\ddot{\phi}_{ij} = \dot{A}_{ij}\dot{q}_{ij} + A_{ij}F_{n_{ij}} + A_{ij}F_{c_{ij}} + A_{ij}(\Delta_{ij} - F_{\mathrm{ad}_{ij}}) \tag{30}$$

Ignoring the uncertainties in the constraint dynamics, we use the following nominal constraint dynamics to derive the constraint forces:

$$\ddot{\phi}_{ij}^{\mathrm{nom}} = \dot{A}_{ij}\dot{q}_{ij} + A_{ij}F_{n_{ij}} + A_{ij}F_{c_{ij}} \tag{31}$$

Setting Eq. (31) to be zero, the constraint force satisfies the following equation:

$$A_{ij}(q_{ij})F_{c_{ij}} = -\dot{A}_{ij}(q_{ij}, \dot{q}_{ij})\dot{q}_{ij} - A_{ij}(q_{ij})F_{n_{ij}} \tag{32}$$

Equation (32) alone does not uniquely determine the constraint force, because we have only one equation and six unknowns (the six components of $F_{c_{ij}}$). The widely used procedure in dynamics to obtain the constraint forces uses the principle of virtual work [16], which states that the constraint forces do not add or remove energy. Therefore, to ensure that the constraint force does no work, we require that $F_{c_{ij}}^T \dot{q}_{ij}$ be zero for every $\dot{q}_{ij}$ satisfying $\dot{\phi}_{ij}(q_{ij}, \dot{q}_{ij}) = 0$, that is,

$$F_{c_{ij}}^T \dot{q}_{ij} = 0, \quad \forall\ \dot{q}_{ij} \in \{\dot{q}_{ij} | A_{ij}(q_{ij})\dot{q}_{ij} = 0\} \tag{33}$$

From Eq. (33), it is clear that $F_{c_{ij}}$ must be orthogonal to the velocity vector $\dot{q}_{ij}$. Since $\dot{q}_{ij}$ must lie in the null space of $A_{ij}(q_{ij})$, the constraint force $F_{c_{ij}}$ must lie in the null space complement of $A_{ij}(q_{ij})$. Thus, the vector $F_{c_{ij}}$ satisfying Eq. (33) can be expressed in the form

$$F_{c_{ij}} = A_{ij}^T(q_{ij})\lambda_{ij} \tag{34}$$

where λ_{ij} is the Lagrange multiplier, which is obtained by substituting Eq. (34) into Eq. (32):

$$A_{ij}(q_{ij})A_{ij}^T(q_{ij})\lambda_{ij} = -\dot{A}_{ij}(q_{ij}, \dot{q}_{ij})\dot{q}_{ij} - A_{ij}(q_{ij})F_{n_{ij}} \tag{35}$$

The constraint force for aircraft i and j are then given by $F_{c_i} = [I_3, \emptyset_3]F_{c_{ij}}$ and $F_{c_j} = [\emptyset_3, I_3]F_{c_{ij}}$ where I_3 and $\emptyset_3$ are the identity and zero matrices, respectively, of dimension three.

The entire discussion on the development of the constraint forces so far was based on the assumption that, at the start of the motion of the aircraft, the constraint equations are satisfied. To consider arbitrary initial conditions for the aircraft, which do not satisfy the constraint equations, we will use the notion of feedback in the constraint acceleration equation. This will account for the mismatch in the initial condition and appropriately compute the constraint force. A similar idea was used to prevent numerical drift in the simulation of dynamic equations with constraints [25]. In the application of cooperative control of a group of aircraft, we generally require the desired formation of the group as well as navigation. Instead of solving for $\ddot{\phi}_{ij}^{\mathrm{nom}} = 0$ to determine the constraint force [18], the following equation will be used:

$$\ddot{\phi}_{ij}^{\mathrm{nom}} = -k_d\dot{\phi}_{ij} - k\lambda_{ij} \tag{36}$$

where k_d and k are positive constants. Therefore, combining Eqs. (31) and (36) and simplifying using Eq. (34), the constraint force vector for the two aircraft can be calculated based on the following equations:

$$F_{c_{ij}} = A_{ij}^T(q_{ij})\lambda_{ij} \tag{37}$$

$$\lambda_{ij} = \frac{1}{k + A_{ij}(q_{ij})A_{ij}^T(q_{ij})}[-\dot{A}_{ij}(q_{ij}, \dot{q}_{ij})\dot{q}_{ij} - A_{ij}(q_{ij})F_{n_{ij}} - k_d\dot{\phi}_{ij}(q_{ij}, \dot{q}_{ij})] \tag{38}$$

The constraint forces for aircraft i and j are then given by $F_{c_i} = [I_3, \emptyset_3]F_{c_{ij}}$ and $F_{c_j} = [\emptyset_3, I_3]F_{c_{ij}}$. Note that the constraint forces on the two aircraft satisfy $F_{c_i} = -F_{c_j}$. Because they are internal forces, addition of the ith and jth dynamics will result in cancellation of these forces for the two-aircraft case.

We consider the following Lyapunov function candidate

$$E_{ij} = \tfrac{1}{2}kc_1 e_{ij}^T e_{ij} + \tfrac{1}{2}k\dot{e}_{ij}^T\dot{e}_{ij} + \tfrac{1}{2}\dot{\phi}_{ij}^2 + \tfrac{1}{2}\Gamma\tilde{C}_{D_{0ij}}^T\tilde{C}_{D_{0ij}} \tag{39}$$

where Γ is a positive constant, $e_{ij} = [e_i^T, e_j^T]^T \in \mathbb{R}^6$, and $\tilde{C}_{D_{0ij}} = [\tilde{C}_{D_{0i}}, \tilde{C}_{D_{0j}}]^T \in \mathbb{R}^2$ with $\tilde{C}_{D_{0i}} = \hat{C}_{D_{0i}} - C_{D_{0i}}$ and $\tilde{C}_{D_{0j}} = \hat{C}_{D_{0j}} - C_{D_{0j}}$. The derivative of E_{ij} with respect to time is given by

$$\dot{E}_{ij} = kc_1\dot{e}_{ij}^T e_{ij} + k\dot{e}_{ij}^T\ddot{e}_{ij} + \dot{\phi}_{ij}\ddot{\phi}_{ij} + \Gamma\tilde{C}_{D_{0ij}}^T\dot{\hat{C}}_{D_{0ij}}$$

From Eqs. (30), (31), and (36), we have

$$\ddot{\phi}_{ij} = -k_d\dot{\phi}_{ij} - k\lambda_{ij} + A_{ij}(\Delta_{ij} - F_{\mathrm{ad}_{ij}}) \tag{40}$$

Substituting Eqs. (29) and (40), and recognizing that $\ddot{e}_{ij} = \ddot{q}_{ij} - \ddot{q}_{d_{ij}}$, we obtain

$$\begin{aligned}\dot{E}_{ij} &= kc_1\dot{e}_{ij}^T e_{ij} + k\dot{e}_{ij}^T[F_{n_{ij}} + F_{c_{ij}} + (\Delta_{ij} - F_{\mathrm{ad}_{ij}}) - \ddot{q}_{d_{ij}}] \\ &\quad + \dot{\phi}_{ij}[-k_d\dot{\phi}_{ij} - k\lambda_{ij} + A_{ij}(\Delta_{ij} - F_{\mathrm{ad}_{ij}})] + \Gamma\tilde{C}_{D_{0ij}}^T\dot{\hat{C}}_{D_{0ij}} \\ &= kc_1\dot{e}_{ij}^T e_{ij} + k\dot{e}_{ij}^T(F_{n_{ij}} + F_{c_{ij}} - \ddot{q}_{d_{ij}}) + \dot{\phi}_{ij}(-k_d\dot{\phi}_{ij} - k\lambda_{ij}) \\ &\quad + (k\dot{e}_{ij}^T + \dot{\phi}_{ij}A_{ij})(\Delta_{ij} - F_{\mathrm{ad}_{ij}}) + \Gamma\tilde{C}_{D_{0ij}}^T\dot{\hat{C}}_{D_{0ij}}\end{aligned}$$

Substitution of $F_{n_{ij}}$ and $F_{c_{ij}}$ expressions from Eqs. (20) and (34), and noting that $\dot{q}_{d_{ij}} = [\dot{q}_{d_i}^T, \dot{q}_{d_j}^T]^T$ is the desired navigation velocity, which must satisfy $\dot{q}_{d_{ij}}^T A_{ij}^T = 0$, we have

$$\begin{aligned}\dot{E}_{ij} &= kc_1\dot{e}_{ij}^T e_{ij} + k\dot{e}_{ij}^T(\ddot{q}_{d_{ij}} - c_1 e_{ij} - c_2\dot{e}_{ij} + A_{ij}^T\lambda_{ij} - \ddot{q}_{d_{ij}}) \\ &\quad + \dot{\phi}_{ij}(-k_d\dot{\phi}_{ij} - k\lambda_{ij}) + (k\dot{e}_{ij}^T + \dot{\phi}_{ij}A_{ij})(\Delta_{ij} - F_{\mathrm{ad}_{ij}}) \\ &\quad + \Gamma\tilde{C}_{D_{0ij}}^T\dot{\hat{C}}_{D_{0ij}} \\ &= -kc_2\dot{e}_{ij}^T\dot{e}_{ij} - k_d\dot{\phi}_{ij}^2 + k\dot{q}_{ij}^T A_{ij}^T\lambda_{ij} - \dot{q}_{d_{ij}}A_{ij}^T\lambda_{ij} + A_{ij}\dot{q}_{ij}(-k\lambda_{ij}) \\ &\quad + (k\dot{e}_{ij}^T + \dot{\phi}_{ij}A_{ij})(\Delta_{ij} - F_{\mathrm{ad}_{ij}}) + \Gamma\tilde{C}_{D_{0ij}}^T\dot{\hat{C}}_{D_{0ij}} \\ &= -kc_2\dot{e}_{ij}^T\dot{e}_{ij} - k_d\dot{\phi}_{ij}^2 + (k\dot{e}_{ij}^T + \dot{\phi}_{ij}A_{ij})(\Delta_{ij} - F_{\mathrm{ad}_{ij}}) \\ &\quad + \Gamma\tilde{C}_{D_{0ij}}^T\dot{\hat{C}}_{D_{0ij}}\end{aligned}$$

Note that A_{ij} can be expressed as $A_{ij} = [a_{ij}^T, a_{ji}^T]$ with $a_{ij} = -a_{ji}$, then

$$\begin{aligned}\dot{E}_{ij} &= -kc_2\dot{e}_{ij}^T\dot{e}_{ij} - k_d\dot{\phi}_{ij}^2 + (k\dot{e}_{ij}^T + \dot{\phi}_{ij}A_{ij})\begin{bmatrix}-\Psi_i\tilde{C}_{D_{0i}} \\ -\Psi_j\tilde{C}_{D_{0j}}\end{bmatrix} \\ &\quad + \Gamma\tilde{C}_{D_{0ij}}^T\dot{\hat{C}}_{D_{0ij}} \\ &= -kc_2\dot{e}_{ij}^T\dot{e}_{ij} - k_d\dot{\phi}_{ij}^2 + [\,k\dot{e}_i^T + \dot{\phi}_{ij}a_{ij}^T \quad k\dot{e}_i^T + \dot{\phi}_{ij}a_{ji}^T\,] \\ &\quad \times \begin{bmatrix}-\Psi_i\tilde{C}_{D_{0i}} \\ -\Psi_j\tilde{C}_{D_{0j}}\end{bmatrix} + \Gamma[\,\tilde{C}_{D_{0i}}^T \quad \tilde{C}_{D_{0j}}^T\,]\begin{bmatrix}\dot{\hat{C}}_{D_{0i}} \\ \dot{\hat{C}}_{D_{0j}}\end{bmatrix} \\ &= -kc_2\dot{e}_{ij}^T\dot{e}_{ij} - k_d\dot{\phi}_{ij}^2 + [\,\tilde{C}_{D_{0i}}^T \quad \tilde{C}_{D_{0j}}^T\,] \\ &\quad \times \begin{bmatrix}-k\Psi_i^T\dot{e}_i - \dot{\phi}_{ij}\Psi_i^T a_{ij} + \Gamma\dot{\hat{C}}_{D_{0i}} \\ -k\Psi_j^T\dot{e}_j - \dot{\phi}_{ij}\Psi_j^T a_{ji} + \Gamma\dot{\hat{C}}_{D_{0j}}\end{bmatrix}\end{aligned} \tag{41}$$

To estimate the unknown parameters $C_{D_{0i}}$ and $C_{D_{0j}}$, we use the gradient projection algorithm [26]. Consider a convex parameter set Π_i given by

$$\hat{C}_{D_{0i}} \in \Pi_i \Longleftrightarrow |\hat{C}_{D_{0i}} - \rho_i| < \sigma_i \tag{42}$$

with ρ_i and σ_i some given real numbers. Consider the function

$$\mathcal{P}_i(\hat{C}_{D_{0i}}) = \frac{2}{\epsilon_i}\left[\left|\frac{\hat{C}_{D_{0i}} - \rho_i}{\sigma_i}\right|^q - 1 + \epsilon_i\right] \tag{43}$$

where $0 < \epsilon_i < 1$ and $q \geq 2$. Consider the "smooth projection" $\mathrm{Proj}(\cdot)$, which will be used to estimate $\hat{C}_{D_{0i}}$ while maintaining it in Π_i:

$$\mathrm{Proj}\,(\hat{C}_{D_{0i}}, y_i) = \begin{cases} y_i, & \text{if } \mathcal{P}_i < 0 \\ y_i, & \text{if } \mathcal{P}_i = 0 \text{ and } \nabla_{\mathcal{P}_i}^T y_i \leq 0 \\ y_i - \frac{\mathcal{P}_i\nabla_{\mathcal{P}_i}\nabla_{\mathcal{P}_i}^T}{\|\nabla_{\mathcal{P}_i}\|^2}y_i, & \text{otherwise}\end{cases} \tag{44}$$

where

$$\nabla_{\mathcal{P}_i} = \left[\frac{\partial\mathcal{P}_i(\hat{C}_{D_{0i}})}{\partial\hat{C}_{D_{0i}}}\right]^T$$

is a column vector. Based on the smooth projection as just defined, $\hat{C}_{D_{0i}}$ is estimated by

$$\begin{aligned}\dot{\hat{C}}_{D_{0i}} &= \Gamma^{-1}\mathrm{Proj}(\hat{C}_{D_{0i}}, k\Psi_i^T\dot{e}_i + \dot{\phi}_{ij}\Psi_i^T a_{ij}) \\ &= \Gamma^{-1}\mathrm{Proj}(\hat{C}_{D_{0i}}, k\Psi_i^T\dot{e}_i + \dot{\phi}_{ij}\Psi_i^T[I_3, \emptyset_3]A_{ij}^T)\end{aligned} \tag{45}$$

and $\hat{C}_{D_{0j}}$ is estimated by

$$\begin{aligned}\dot{\hat{C}}_{D_{0j}} &= \Gamma^{-1}\mathrm{Proj}(\hat{C}_{D_{0j}}, k\Psi_j^T\dot{e}_j + \dot{\phi}_{ji}\Psi_j^T a_{ji}) \\ &= \Gamma^{-1}\mathrm{Proj}(\hat{C}_{D_{0j}}, k\Psi_i^T\dot{e}_i + \dot{\phi}_{ji}\Psi_i^T[I_3, \emptyset_3]A_{ji}^T)\end{aligned} \tag{46}$$

where the corresponding projection for the jth aircraft is given by replacing the index i by j in Eqs. (42–44). Substitution of the adaptation law in Eq. (41) gives

$$\dot{E}_{ij} = -kc_2\dot{e}_{ij}^T\dot{e}_{ij} - k_d\dot{\phi}_{ij}^2 \leq 0 \tag{47}$$

Therefore, E_{ij} is a Lyapunov function. As a result, e_{ij}, $\dot{e}_{ij}$, $\dot{\phi}_{ij}$, and $\tilde{C}_{D_{0ij}}$ are bounded. From Eqs. (39) and (47), we can conclude that $\dot{e}_{ij}$ and $\dot{\phi}_{ij}$ are square integrable signals. Further, from the dynamics of the constraint and the tracking error,

$$\ddot{\phi}_{ij} = -k_d\dot{\phi}_{ij} - k\lambda_{ij} - A_{ij}\begin{bmatrix}\Psi_i\tilde{C}_{D_{0i}} \\ \Psi_j\tilde{C}_{D_{0j}}\end{bmatrix} \tag{48}$$

$$\ddot{e}_{ij} = -c_1 e_{ij} - c_2 \dot{e}_{ij} + A_{ij}^T \lambda_{ij} - \begin{bmatrix} \Psi_i \tilde{C}_{D_{0i}} \\ \Psi_j \tilde{C}_{D_{0j}} \end{bmatrix} \tag{49}$$

we can conclude that both $\ddot{\phi}_{ij}$ and $\ddot{e}_{ij}$ are bounded. Therefore, $\dot{e}_{ij}$ and $\dot{\phi}_{ij}$ converge to zero asymptotically by invoking Barbalat's lemma. Further, we can show via direct calculation that $\dddot{\phi}_{ij}$ and $\dddot{e}_{ij}$ are bounded by differentiating Eqs. (48) and (49). Therefore, the signals $\ddot{\phi}_{ij}$ and $\ddot{e}_{ij}$ asymptotically converge to zero. From Eqs. (48) and (49), we can conclude that e_{ij}, $\tilde{C}_{D_{0i}}$, and $\tilde{C}_{D_{0j}}$ converge to the relationship given by

$$e_{ij} = -\frac{1}{c_1}\left(\frac{I_6 + A_{ij}^T A_{ij}}{k}\right)\begin{bmatrix} \Psi_i \tilde{C}_{D_{0i}} \\ \Psi_j \tilde{C}_{D_{0j}} \end{bmatrix} \tag{50}$$

Therefore, as expected, the position tracking error depends on the uncertain parameter estimation error. The error can be decreased by increasing c_1. Moreover, from the definition of the constraint vector,

$$\begin{aligned} \phi_{ij}(q_{ij}) &= \| q_i - q_j \| - d_{ij} = \| (e_i - e_j) + (q_{d_i} - q_{d_j}) \| - d_{ij} \\ &\le \| e_i - e_j \| \end{aligned} \tag{51}$$

since $\| q_{d_i} - q_{d_j} \| = d_{ij}$. Note that e_i and e_j are the first and last three elements, respectively, of e_{ij}. Equation (51) indicates a relationship between the tracking errors and the ability to maintain the distance constraint for a pair of communicating aircraft; this is reasonable and should be expected because any tracking error will result in an error in keeping the desired distance between the two aircraft and vice versa.

B. Distributed Control Algorithm for Multiple Aircraft

As in the previous section, we consider the applied force F_i acting on the ith aircraft to be given by Eq. (19), with the navigational feedback control F_{n_i} given by Eq. (20), and the adaptive control F_{ad_i} given by Eq. (21). Taking into consideration the communication between different aircraft in the group as per the information flow graph, the adaptation law for the drag coefficient is given by

$$\dot{\hat{C}}_{D_{0i}} = \Gamma^{-1}\mathrm{Proj}\left(k\Psi_i^T \dot{e}_i + \sum_{(i,j)\in\mathcal{E}} \dot{\phi}_{ij} \Psi_i^T [I_3, \emptyset_3] A_{ij}^T\right) \tag{52}$$

The constraint force acting on the ith aircraft is chosen as the total constraint force contribution from all aircraft that directly communicate with it, that is, F_{c_i} is given by

$$F_{c_i} = \sum_{(i,j)\in\mathcal{E}} [I_3, \emptyset_3] A_{ij}^T \lambda_{ij} \tag{53}$$

$$\lambda_{ij} = \frac{1}{k + A_{ij}A_{ij}^T}(-\dot{A}_{ij}\dot{q}_{ij} - A_{ij}F_{n_{ij}} - k_d \dot{\phi}_{ij}) \tag{54}$$

Notice that the choice of the total constraint force of this form allows addition and/or removal of aircraft from the group by simply adding and/or removing the constraint force term corresponding to the vehicle added/removed from the group. Therefore, the proposed constraint force algorithm is scalable, that is, the complexity of the constraint force grows gracefully with the number of vehicles and the complexity of the communication graph $\mathcal{E}$.

To show that the proposed control input tracks the desired navigation trajectories and achieves, and maintains, the given formation, we consider the following composite Lyapunov function candidate:

$$\begin{aligned} E &= \frac{1}{2}\sum_{i=1}^n k c_1 e_i^T e_i + \frac{1}{2}\sum_{i=1}^n k \dot{e}_i^T \dot{e}_i \\ &+ \frac{1}{2}\sum_{i=1}^n \sum_{(i,j)\in\mathcal{E},j>i} \dot{\phi}_{ij}^2 + \frac{1}{2}\sum_{i=1}^n \Gamma \tilde{C}_{D_{0i}}^2 \end{aligned} \tag{55}$$

The derivative of E with respect to time is given by

$$\begin{aligned} \dot{E} &= k c_1 \sum_{i=1}^n \dot{e}_i^T e_i + k\sum_{i=1}^n \dot{e}_i^T \ddot{e}_i + \sum_{i=1}^n \sum_{(i,j)\in\mathcal{E},j>i} \dot{\phi}_{ij}\ddot{\phi}_{ij} \\ &+ \sum_{i=1}^n \Gamma \tilde{C}_{D_{0i}} \dot{\tilde{C}}_{D_{0i}} \\ &= k c_1 \sum_{i=1}^n \dot{e}_i^T e_i + k\sum_{i=1}^n \dot{e}_i^T (\ddot{q}_i - \ddot{q}_{d_i}) \\ &+ \sum_{i=1}^n \sum_{(i,j)\in\mathcal{E},j>i} \dot{\phi}_{ij}(\dot{A}_{ij}\dot{q}_{ij} + A_{ij}\ddot{q}_{ij}) + \sum_{i=1}^n \Gamma \tilde{C}_{D_{0i}} \dot{\tilde{C}}_{D_{0i}} \end{aligned}$$

Substituting the control law (18) and the dynamics of the aircraft (16) into $\dot{E}$, and simplifying, we get

$$\begin{aligned} \dot{E} &= k c_1 \sum_{i=1}^n \dot{e}_i^T e_i + k\sum_{i=1}^n \dot{e}_i^T (F_i + F_{c_i} - \ddot{q}_{d_i}) \\ &+ \sum_{i=1}^n \sum_{(i,j)\in\mathcal{E},j>i} \dot{\phi}_{ij}(\dot{A}_{ij}\dot{q}_{ij} + A_{ij}F_{n_{ij}} + A_{ij}F_{c_{ij}}) \\ &+ \sum_{i=1}^n \sum_{(i,j)\in\mathcal{E},j>i} \dot{\phi}_{ij}A_{ij}(\Delta_{ij} - F_{\mathrm{ad}_{ij}}) + k\sum_{i=1}^n \dot{e}_i^T(\Delta_i - F_{\mathrm{ad}_i}) \\ &+ \sum_{i=1}^n \Gamma \tilde{C}_{D_{0i}} \dot{\tilde{C}}_{D_{0i}} \end{aligned}$$

Upon simplification, we can write $\dot{E}$ as

$$\begin{aligned} \dot{E} &= -k c_2 \sum_{i=1}^n \dot{e}_i^T \dot{e}_i - k_d \sum_{i=1}^n \sum_{(i,j)\in\mathcal{E},j>i} \dot{\phi}_{ij}^2 \\ &+ k\left(\sum_{i=1}^n \dot{q}_i^T F_{c_i} - \sum_{i=1}^n \sum_{(i,j)\in\mathcal{E},j>i} \dot{\phi}_{ij}\lambda_{ij}\right) - k\sum_{i=1}^n \dot{q}_{d_i}^T F_{c_i} \\ &- \sum_{i=1}^n \sum_{(i,j)\in\mathcal{E},j>i} \dot{\phi}_{ij}[a_{ij}^T \quad a_{ji}^T]\begin{bmatrix} \Psi_i \tilde{C}_{D_{0i}} \\ \Psi_j \tilde{C}_{D_{0j}} \end{bmatrix} \\ &- k\sum_{i=1}^n \dot{e}_i^T \Psi_i \tilde{C}_{D_{0i}} + \sum_{i=1}^n \Gamma \tilde{C}_{D_{0i}} \dot{\tilde{C}}_{D_{0i}} \end{aligned} \tag{56}$$

In Eq. (56), we can show that the third and fourth terms are identically equal to zero. Note that

$$\begin{aligned} &\sum_{i=1}^n \dot{q}_i^T F_{c_i} - \sum_{i=1}^n \sum_{(i,j)\in\mathcal{E},j>i} \dot{\phi}_{ij}\lambda_{ij} \\ &= \sum_{i=1}^n \dot{q}_i^T \sum_{(i,j)\in\mathcal{E}} [I_3, \emptyset_3] A_{ij}^T \lambda_{ij} - \sum_{i=1}^n \sum_{(i,j)\in\mathcal{E},j>i} A_{ij}\dot{q}_{ij}\lambda_{ij} \\ &= \sum_{i=1}^n \sum_{(i,j)\in\mathcal{E}} \dot{q}_i^T a_{ij}\lambda_{ij} - \sum_{i=1}^n \sum_{(i,j)\in\mathcal{E},j>i} (a_{ij}^T \dot{q}_i + a_{ji}^T \dot{q}_j)\lambda_{ij} \end{aligned}$$

Since $\lambda_{ij} = \lambda_{ji}$, we have

$$\sum_{i=1}^n \sum_{(i,j)\in\mathcal{E},j>i} (a_{ij}^T \dot{q}_i + a_{ji}^T \dot{q}_j)\lambda_{ij} = \sum_{i=1}^n \sum_{(i,j)\in\mathcal{E}} a_{ij}^T \dot{q}_i \lambda_{ij}$$

Thus, we have

$$\sum_{i=1}^n \dot{q}_i^T F_{c_i} - \sum_{i=1}^n \sum_{(i,j)\in\mathcal{E},j>i} \dot{\phi}_{ij}\lambda_{ij} = 0 \tag{57}$$

Further, since $\dot{q}_{d_i}$ is the desired velocity, it must satisfy $\dot{q}_{d_{ij}}^T A_{ij}^T = 0$ for any $i \neq j$. Hence,

$$\sum_{i=1}^{n} \dot q_{d_i}^T F_{c_i} = \sum_{i=1}^{n} \sum_{(i,j)\mathcal{E}j>i} \dot q_{d_{ij}}^T A_{ij}^T \lambda_{ij} = 0$$

Therefore,

$$\begin{aligned}\dot E = & -kc_2 \sum_{i=1}^{n} \dot e_i^T \dot e_i - k_d \sum_{i=1}^{n} \sum_{(i,j)\in\mathcal{E}, j>i} \dot\phi_{ij}^2 \\ & - \sum_{i=1}^{n} \sum_{(i,j)\in\mathcal{E}, j>i} (\dot\phi_{ij} \tilde C_{D_{0i}}^T \Psi_i^T a_{ij} + \dot\phi_{ji} \tilde C_{D_{0j}}^T \Psi_j^T a_{ji}) \\ & - k \sum_{i=1}^{n} \tilde C_{D_{0i}}^T \Psi_i^T \dot e_i + \sum_{i=1}^{n} \Gamma \tilde C_{D_{0i}} \dot{\tilde C}_{D_{0i}} \\ = & -kc_2 \sum_{i=1}^{n} \dot e_i^T \dot e_i - k_d \sum_{i=1}^{n} \sum_{(i,j)\in\mathcal{E}, j>i} \dot\phi_{ij}^2 - \sum_{i=1}^{n} \sum_{(i,j)\in\mathcal{E}} \dot\phi_{ij} \tilde C_{D_{0i}}^T \Psi_i^T a_{ij} \\ & - k \sum_{i=1}^{n} \tilde C_{D_{0i}}^T \Psi_i^T \dot e_i + \sum_{i=1}^{n} \Gamma \tilde C_{D_{0i}} \dot{\tilde C}_{D_{0i}} \end{aligned} \tag{58}$$

Using the adaptive law (52), we have

$$\dot E = -kc_2 \sum_{i=1}^{n} \dot e_i^T \dot e_i - k_d \sum_{i=1}^{n} \sum_{(i,j)\in\mathcal{E}, j>i} \dot\phi_{ij}^2 \le 0 \tag{59}$$

Using the same arguments as in the previous section, we can conclude that all signals are bounded, the signals $\dot e_i$ and $\dot\phi_{ij}$ asymptotically converge to zero, and e_i and ϕ_{ij} are bounded by a function of the parameter estimation error, similar to Eq. (50). The results of this section are summarized in the following theorem.

Theorem 1: For a group of aircraft given by the dynamics (1–6), the choice of the following control algorithm

$$\mu_{c_i} = \arctan\left(\frac{U_{y_i} \cos\chi_i - U_{x_i} \sin\chi_i}{\cos\gamma_i (U_{h_i} + g) - \sin\gamma_i (U_{x_i} \cos\chi_i + U_{y_i} \sin\chi_i)}\right) \tag{60}$$

$$L_{c_i} = m_i \frac{\cos\gamma_i (U_{h_i} + g) - \sin\gamma_i (U_{x_i} \cos\chi_i + U_{y_i} \sin\chi_i)}{\cos\mu_{c_i}} \tag{61}$$

$$\begin{aligned} T_{c_i} = & \, m_i [\sin\gamma_i (U_{h_i} + g) + \cos\gamma_i (U_{x_i} \cos\chi_i + U_{y_i} \sin\chi_i)] \\ & + 2K_i \frac{L_{c_i}^2}{\rho V_i^2 S_i} \end{aligned} \tag{62}$$

$$\begin{bmatrix} U_{x_i} \\ U_{y_i} \\ U_{h_i} \end{bmatrix} = F_{n_i} - F_{\mathrm{ad}_i} + F_{c_i} \tag{63}$$

$$F_{n_i} = \ddot q_{d_i} - c_1 e_i - c_2 \dot e_i \tag{64}$$

$$F_{\mathrm{ad}_i} = \Psi_i \hat C_{D_{0i}} \tag{65}$$

$$\dot{\hat C}_{D0i} = \Gamma^{-1} \mathrm{Proj}\left(k \Psi_i^T \dot e_i + \sum_{(i,j)\in\mathcal{E}} \dot\phi_{ij} \Psi_i^T [I_3, \emptyset_3] A_{ij}^T\right) \tag{66}$$

$$F_{c_i} = \sum_{(i,j)\in\mathcal{E}} \frac{[I_3, \emptyset_3] A_{ij}^T}{k + A_{ij} A_{ij}^T} (-\dot A_{ij} \dot q_{ij} - A_{ij} F_{n_{ij}} - k_d \dot\phi_{ij}) \tag{67}$$

will ensure that all signals are bounded, the signals $\dot e_i$ and $\dot\phi_{ij}$ asymptotically converge to zero.

Remark 1: The inner-loop variables to be controlled are the lift L_i (or angle of attack α_i), bank angle μ_i, and sideslip angle β_i, whereas the output variables of the controller are the control surface deflections of aileron, elevator, and rudder, δ_{a_i}, δ_{e_i}, and δ_{r_i}. From the aircraft equations of motion, $\dot L_i$ and $\dot\mu_i$ are functions of the body-axis angular velocities p_i, q_i, and r_i. Considering an additional output $\dot\beta_i$ for control, the required angular velocities can be determined by solving the equations which relate $\dot L_i$, $\dot\mu_i$, $\dot\beta_i$ to p_i, q_i, r_i. Then, using the timescale separation and singular perturbation theory, one can design the inner-loop to obtain the control surface deflection of aileron, elevation, and rudder, δ_{a_i}, δ_{e_i}, and δ_{r_i}.

Remark 2: Safe distance between two communicating aircraft can be maintained by modifying the distance constraint function. This is achieved by modifying each distance constraint function to the following form:

$$\phi_{ij} = \frac{\| q_i - q_j \| - d_{ij}}{\| q_i - q_j \| - r_s}, \quad \forall\ (i,j) \in \mathcal{E} \tag{68}$$

where r_s denotes the safe distance between any two communicating aircraft. The derivative of ϕ_{ij} with respect to time is given by

$$\dot\phi_{ij} = \frac{(d_{ij} - r)(q_i - q_j)^T (\dot q_i - \dot q_j)}{(\| q_i - q_j \| - r_s)^2 \, \| q_i - q_j \|} \tag{69}$$

Since $\dot\phi_{ij}$ is bounded and

$$\lim_{\| q_i - q_j \| \to r_s +} \dot\phi_{ij}(q_{ij}, \dot q_{ij}) = \infty, \quad \forall\ (i,j) \in \mathcal{E} \tag{70}$$

any pair of communicating aircraft will never enter the unsafe region given by $\Omega = \{q_{ij}\colon \| q_i - q_j \| \le r_s\}$. Note that it is assumed that the initial distance between any two communicating aircraft is larger than the safe distance.

IV. Simulations

The performance of three aircraft flying in a V formation along a straight path is evaluated. The distributed control law given in Theorem 1 is applied. Each aircraft in the group starts from an arbitrary location that does not satisfy the constraint equations. The desired formation is defined as a V shape with the length of the edges equal to 100 m and at the same height of 300 m. The desired navigation trajectory for each vehicle within the V formation is taken as a straight line with a constant velocity of 40 m/s. The information flow graph is defined as aircraft 3 communicating with aircraft 1 and 2, whereas there is no communication between aircraft 1 and 2. The

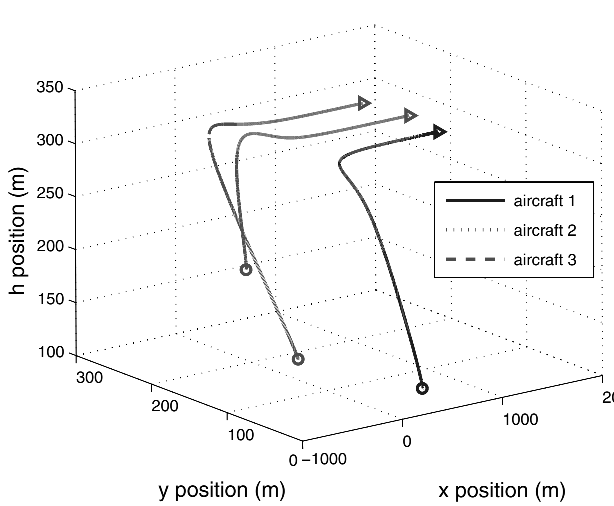

Fig. 3 Three-dimensional view of triangle formation.

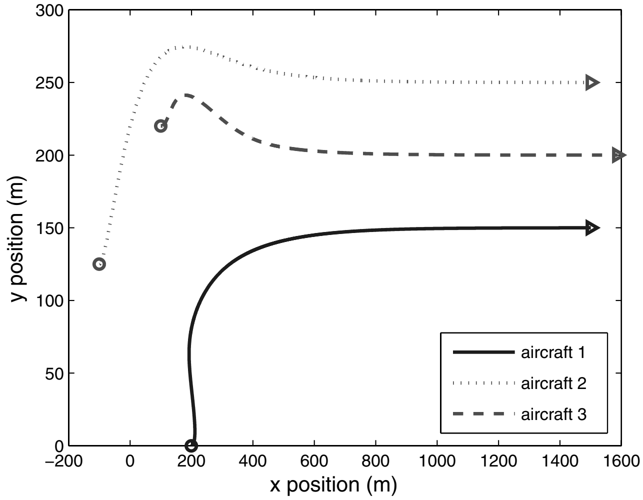

Fig. 4 Projection onto *xy* plane of triangle formation.

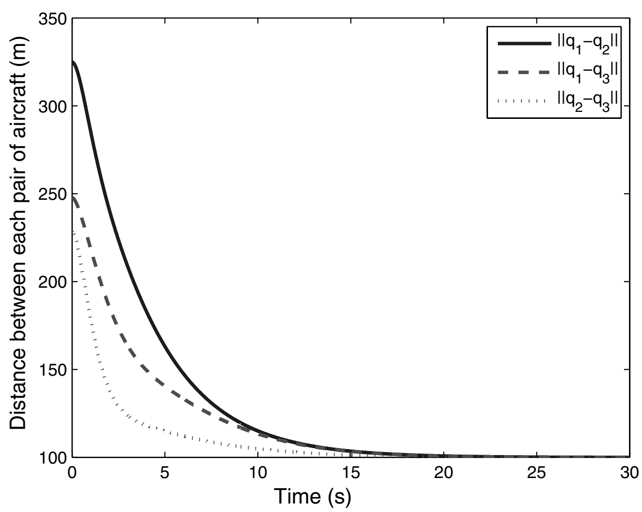

Fig. 5 Distance between pairs of aircraft.

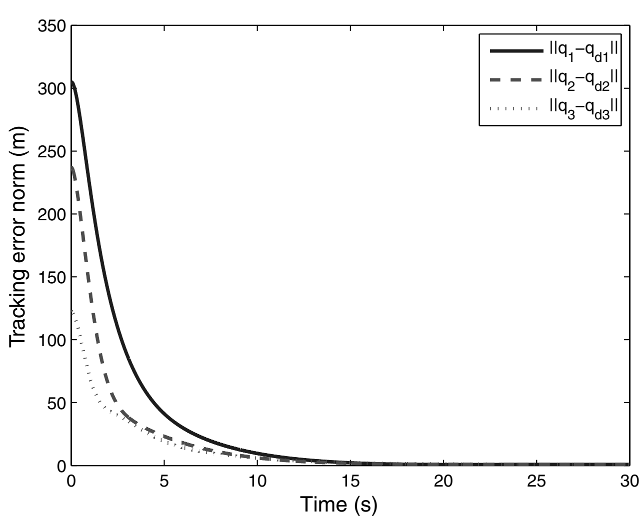

Fig. 6 Tracking error.

initial positions of the three aircraft are given by $q_1(0) = [200, 0, 125]^T$ m, $q_2(0) = [-100, 125, 125]^T$ m, and $q_3(0) = [100, 220, 180]^T$ m, and initial heading angles and flight-path angles are zero. The drag coefficients are $C_{D_{0i}} = 0.02$ and $K_i = 0.25$ for each aircraft. To evaluate the performance of the adaptive controller, the initial value of the drag coefficient estimate $\hat{C}_{D_{0i}}$ is taken as 0.016, which reflects as a 20% uncertainty on the true value. The adaptation gain value is chosen as $\Gamma = 10,000$. The lower and upper bounds on the parameter for the projection algorithm are chosen as 0.014 and 0.026, respectively, and the tolerance is chosen as $\epsilon = 0.25$. The gain parameters for each aircraft in the distributed control algorithm are selected as $c_1 = 2$, $c_2 = 2.8$, $k_d = 2.6$, and $k = 1.8$. The simulation result is shown in a three-dimensional view in Fig. 3 and a projection view onto the xy plane in Fig. 4. Each aircraft in the group starts at the initial position denoted by ∘, and reaches a desired triangle formation while approaching the desired navigation trajectory. The corresponding interaircraft distance is shown in Fig. 5. The tracking error of each aircraft is shown in Fig. 6.

V. Conclusions

Based on the theory of constraint forces in analytical dynamics, a stable, distributed control algorithm for multiple aircraft formation flight has been developed. The nonlinear dynamics of each aircraft is feedback linearized with the output as positions of the center of gravity of the aircraft in an Earth-based reference frame. Given a formation, an information flow pattern, and a desired trajectory, the distributed control algorithm developed for each aircraft in the group is capable of achieving and maintaining the formation along the desired group trajectory. Moreover, the proposed distributed algorithm can also adaptively compensate for the uncertainty in the drag parameter coefficient. Simulation results on an example formation of a group were shown to corroborate the proposed algorithm.

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